Topics offered by Prof. Mertens

Quantum gravity in 3+1 dimensions is hard due to several reasons. One is the non-renormalizability of the action, leading to the path of UV completions such as string theory. A second complication is due to confusions regarding how to think about topics such as causality when the background metric is fluctuating. To circumvent the first problem, we work in lower dimensions where the gravitational path integral does make sense. To tackle the second problem, we can use holography to have an anchoring point (the so-called holographic boundary), where we do understand physics.
Within the resulting set-up (lower-dimensional holography), the last couple of years have seen remarkable progress. Starting with Kitaev's construction of the Sachdev-Ye-Kitaev quantum mechanical models, going through Jackiw-Teitelboim gravity, wormholes and random matrix descriptions, we have learned valuable lessons concerning exact quantum gravity calculations, Hawking's information paradox ...
Most of the thesis topics are related in one way or another to this set of developments, and are hence directly placed in a context that attracts widespread international attention. Our group is actively pursuing goals along the lines of the thesis topics. Finally, all topics contain sufficient freedom for the student to divert focus towards interesting routes discovered along the way.

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1 The bulk dual of the SYK model and de Sitter gravity

The Sachdev-Ye-Kitaev (SYK) model is a quantum mechanical system of N Majorana fermions $\psi_i(t)$, $i = 1..N$, with p-fermion interaction Hamiltonian

$$H = \frac{1}{p!} \sum_{i_1, i_2, \ldots i_p = 1}^{N} J_{i_1 i_2 \ldots i_p} \psi_{i_1} \psi_{i_2} \ldots \psi_{i_p},$$

where the $J_{i_1 i_2 \ldots i_p}$ are random couplings drawn from a Gaussian distribution. After averaging over the random couplings, the resulting model is tractable at large N and was argued by Kitaev to be described at low energies by an emergent 1+1 dimensional model of gravity, with features of black hole solutions exhibiting the maximum bound on chaos. In particular, the low energy gravitational model was identified as the Jackiw-Teitelboim (JT) gravity model, capturing the near-horizon region of nearly extremal black holes. This provides evidence that the full SYK model (without taking the limit to JT) is an exact holographic system. However, the actual bulk gravitational dual of the full SYK model has not been pinned down and remains an important open question.

Recent progress has revealed a tractable all-energy limit of the SYK model, the double-scaled SYK limit (DSSYK), where one simultaneously takes $N \to \infty$ and $p \to \infty$, keeping the ratio p^2/N fixed. In this regime, correlation functions are exactly solvable and are governed by a quantum group symmetry. This in turn provides strong clues that the putative bulk dual spacetime in this limit is itself discrete and non-commutative. An alternative perspective is to identify the bulk dual of DSSYK with a parametric deformation of JT gravity. The classical solutions now exhibit a region between the black hole horizon and the cosmological horizon where the on-shell Ricci curvature can be positively curved. This implies a relation between DSSYK and de Sitter holography. This development has drawn a lot of recent attention, since there currently exist no models of de Sitter (dS) quantum gravity derived from a microscopic perspective.

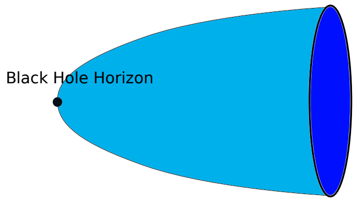
The goals of the student will be to investigate this quantum mechanical model and its double-scaling limit, guided by the first three references. An important emphasis will be on understanding the group-theoretic underpinning. Afterwards, the student is challenged on improving our understanding on the bulk dual description of DSSYK, following recent attention including the last two references. Alternatively, this model is sufficiently unexplored to bend the topic towards other promising avenues, such as extensions of the original SYK model (complex SYK, or adding flavor degrees of freedom etc). Any (partial) advances can already attract attention among the wider community.

References:

https://arxiv.org/abs/1604.07818
https://arxiv.org/abs/2212.13668
https://arxiv.org/abs/2306.00941
https://arxiv.org/abs/2402.00635
https://arxiv.org/abs/2401.05575

2 The 2d black hole in string theory

Finding exact models of black holes is a very important problem in theoretical physics. This is especially true when one can achieve this in a UV-complete theory such as string theory. One of these big examples is the 2d black hole solution in string theory, discovered by Witten, Dijkgraaf, and Verlinde[2] (first two references below) in 1991. Technically this model has a group theoretical origin as the $SL(2,\mathbb{R})/U(1)$ cigar CFT. The cigar geometry describes the Euclidean section of the black hole, as illustrated below:



After an initial study of the basics of string theory, the goal is to first perform a literature survey of the 2d black hole solution and in particular what has been understood over the years. This includes the partition function of free strings moving on such a black hole manifold, its correlation functions describing string scattering processes, or its most recent applications in terms of the black hole string correspondence principle (as explored e.g. in the last reference below). Although its origins are relatively old, it is clear that this model is still giving us much insight in string theory and its relation to black holes. This means that there is the possibility for new and original computations for the student to perform in this field.

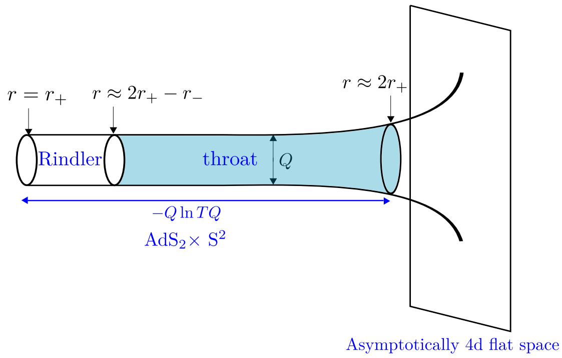
References:

https://inspirehep.net/literature/314576
https://inspirehep.net/literature/315538
https://arxiv.org/abs/hep-th/0202129
https://arxiv.org/abs/2106.02169

3 JT gravity and near-extremal black holes

A big research theme over the past years is the exactly solvable 1+1d Jackiw-Teitelboim (or JT) gravitational model. It is a lower-dimensional model of dilaton gravity for which the gravitational path integral can be exactly computed, which is quite extraordinary. (See the first reference below).

The model also has a large degree of universality attached to it, in the sense that it describes the near-horizon near-extremal black hole dynamics of a large class of higher-dimensional black holes. This includes in particular the rotating Kerr black holes in our universe. The general picture is that the geometry in this regime develops a long throat region close to the black hole horizon whose dynamics is effectively 1+1 dimensional (time and radial), as illustrated in the figure for the Reissner-Nordström black hole:



The goal of this master thesis is to study this near-horizon near-extremal regime of higher-dimensional black hole physics, and in particular to understand how (and when) the JT gravity model controls the physics in this regime.

A particularly interesting research project would be to investigate the relation to the older literature on the Kerr-CFT correspondence. See the second reference for the original paper about the Kerr-CFT correspondence and the last reference for a generalization to Anti-de Sitter and de Sitter space.

References:

https://arxiv.org/abs/2210.10846
https://arxiv.org/abs/0809.4266
https://arxiv.org/pdf/1203.3561
https://arxiv.org/pdf/0811.4393

4 A field theory of multi-universes

Jackiw–Teitelboim (JT) gravity is a universal sector of 3+1-dimensional near-extremal black holes. The partition function of multi-boundary wormholes in JT gravity can be reduced to the problem of calculating volumes in the moduli space of hyperbolic surfaces. Such volumes can be derived via the so-called Mirzakanani's recursion relation which relates hyperbolic surfaces with different topologies. As an integral recursion relation, Mirzaknahi's recursion relation is equivalent to a differential recursion relation on the generating function of connected contributions to the JT partition function. This differential recursion relation is called the Virasoro constraint since it exhibits the famous Virasoro algebra which is the symmetry algebra of 2-dimensional conformal field theory. A natural question is if there is a field theory realization of this specific Virasoro algebra. The answer is the Kodaira-Spencer (KS) theory defined on the complexification of the spectral curve of JT gravity on a disk. KS theory is a quantum theory of trivial deformations of Riemann surfaces. Excitations in KS theory are universes such that Feynman diagrams calculate amplitudes of multi-boundary wormholes. In this sense, KS theory is a closed string field theory.

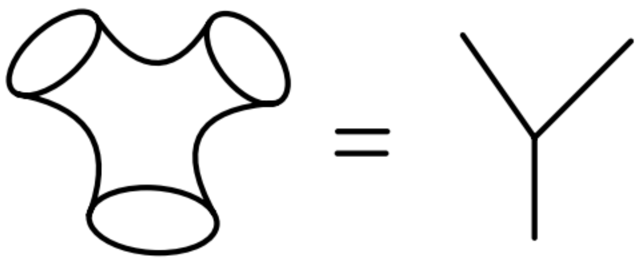


Figure 1: The amplitude of one universe split into two universes is a Feynman diagram with three legs in Kodaira-Spencer theory.

The goal of this master's thesis is to study this universe field theory. From the perspective of string field theory, the holographic duality between JT gravity and random matrix theory is a special case of general open-closed duality. But the open string field theory description in this story is still unclear. Since JT gravity is a dimensional reduction of 3-dimensional pure gravity, one should ask what the dimensional lift of KS theory is. Besides Virasoro symmetry, more symmetries can be added to KS theory as well. After understanding the literature, the above questions can be investigated.

References:

https://arxiv.org/abs/1903.11115
https://arxiv.org/abs/1804.03275
https://arxiv.org/abs/0711.1932
https://arxiv.org/abs/2201.08859